$$\frac{1}{\sqrt{2}}\,\frac{1}{2n+2} \le U_n \le \frac{1}{2n+2}$$

On a : $\begin{cases} \lim\limits_{n\to+\infty} \frac{1}{\sqrt{2}}\,\frac{1}{2n+2} = 0 \\ \lim\limits_{n\to+\infty} \frac{1}{2n+2} = 0 \end{cases}$ d'où $\lim\limits_{n\to+\infty} U_n = 0$

Solution 11 :

$f(x) = x + \sin x \ge 0$ pour $\forall x \in \left[0, \frac{\pi}{2}\right]$

1°/ $\mathcal{A} = \int_0^{\frac{\pi}{2}} (x + \sin x)\,dx = \left[\frac{x^2}{2} - \cos x\right]_0^{\frac{\pi}{2}}$

$\mathcal{A} = \left(\frac{\pi^2}{8} + 1\right)$ unité d'aires .

2°/

a) $\begin{cases} f'(x) = \sin x \\ g(x) = x \end{cases} \Rightarrow \begin{cases} f(x) = -\cos x \\ g'(x) = 1 \end{cases}$

$J = \left[-x\cos x\right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x\,dx$

$J = \left[\sin x\right]_0^{\frac{\pi}{2}} = 1$

b) $V = \pi \int_0^1 f^2(x)\,dx$

$= \pi \int_0^1 (x^2 + \sin^2 x + 2x\sin x)\,dx$

Soit K= $\int_0^{\frac{\pi}{2}} \sin^2 x\,dx = \int_0^{\frac{\pi}{2}} \frac{-\cos 2x + 1}{2}\,dx$

$= \left[\frac{x}{2} - \frac{1}{4}\sin(2x)\right]_0^{\frac{\pi}{2}} = \frac{\pi}{4}$